2021-2022 Jan Exam Solutions

1.

E. False

[2 marks]

SOLUTION 1

S_{amp} and S_{mean} are labelled on the diagram the **wrong way round**.

2.

A. Crack initiation

[2 marks]

3.

A. Bigger

[2 marks]

SOLUTION 3

The yield surface of the Tresca yield criterion is always inside (or equal to) the von Mises equivalent – as shown in the figure below. Therefore to avoid yield (at lower yield stress magnitudes, comparatively), the drive shaft would need to be bigger according to the Tresca yield criteria.

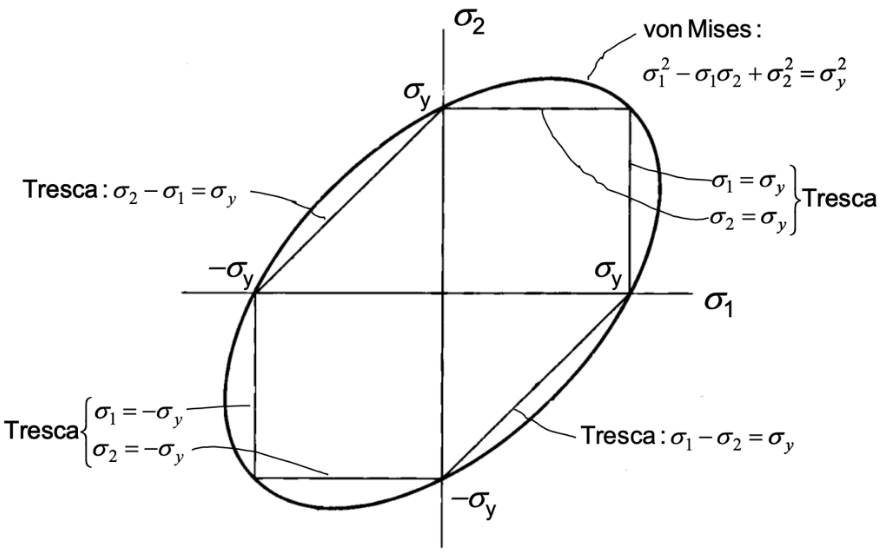


Figure 2.12

4.

A. Hydrostatic Stress

[2 marks]

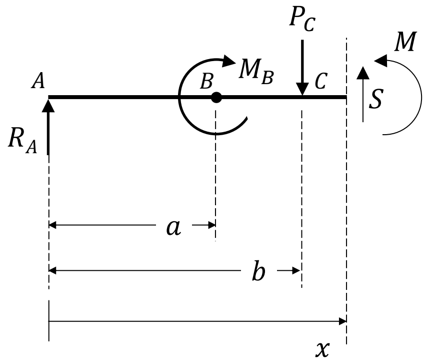
5.

E. $$EI\frac{d^2y}{dx^2} = R_A x + M_B\langle x-a\rangle^0 - P_C\langle x-b\rangle$$

[2 marks]

SOLUTION 5

Sectioning the beam after the last discontinuity (taking origin at left hand side of the beam) and drawing a Free Body Diagram of left-hand side of section:



Taking moments about the section position (and applying Macauley's convention):

$$M + P_C\langle x - b\rangle = R_A x + M_B\langle x - a\rangle^0$$

$$\therefore M = R_A x + M_B\langle x - a\rangle^0 - P_C\langle x - b\rangle$$

Substituting this into the deflection of beams equation:

$$\boldsymbol{EI\frac{d^2y}{dx^2} = R_A x + M_B\langle x - a\rangle^0 - P_C\langle x - b\rangle}$$

6.

E. $\frac{dy}{dx} = \frac{1}{EI}\left(\frac{R_A x^2}{2} + M_B\langle x - a\rangle - \frac{P_C\langle x-b\rangle^2}{2} + A\right)$

[2 marks]

SOLUTION 6

Integrating the 2nd order differential equation from Q5 with respect to x:

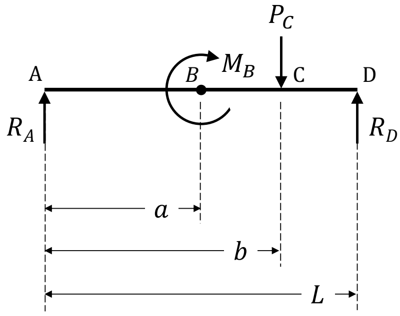
$$\boldsymbol{\frac{dy}{dx} = \frac{1}{EI}\left(\frac{R_A x^2}{2} + M_B\langle x - a\rangle - \frac{P_C\langle x - b\rangle^2}{2} + A\right)}$$

7.

C. $-250\ \mathrm{N}$

[2 marks]

SOLUTION 7



Taking moments about position D:

$$P_C \times (L - b) = R_A \times L + M_B$$

$$\therefore R_A = \frac{P_C(L - b) - M_B}{L}$$

Substituting values of P_C, M_B, L and b gives:

$$R_A = \frac{2000(1 - 0.75) - 750}{1}$$

$$= -250 \text{ N}$$

8.

B. 82.6 MPa

[2 marks]

SOLUTION 8

Axial stress

$$\sigma_a = \frac{F}{A} = \frac{23750}{\pi \times (21.5^2 - 17.75^2)} = \frac{23750}{462.4} = 51.4 \text{ MPa}$$

Torsional shear stress

$$\tau = \frac{Tr}{J} = \frac{32 \times 425 \times 10^3 \times 21.5}{\pi(43^4 - 35.5^4)} = 50.8 \text{ MPa}$$

Centre of Mohr's circle given by:

$$C = \frac{\sigma_a}{2} = \frac{51.4}{2} = 25.7 \text{ MPa}$$

Radius of Mohr's circle, given by:

$$R = \tau_{max} = \sqrt{\left(\frac{\sigma_a}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{51.4}{2}\right)^2 + 50.8^2} = \sqrt{2641.96 + 10{,}322.56} = 56.9 \text{ MPa}$$

Max principal stress given by:

$$\sigma_1 = C + R = 25.7 + 56.9 = \mathbf{82.6\ MPa}$$

9.

B. 4.99×10^{-4} m

[2 marks]

SOLUTION 9

$$\Delta L = L\alpha\Delta T$$

$$\Delta L = 1.9 \times 11 \times 10^{-6} \times 15 = \mathbf{4.99 \times 10^{-4}\ m}$$

10.

C. C

[2 marks]

SOLUTION 10

For cross-section A:

$$I = \frac{bd^3}{12} = \frac{240 \times 240^3}{12} = 276{,}480{,}000 \text{ mm}^4$$

For cross-section B:

$$I = \frac{\pi\left(D_o{}^4 - D_i{}^4\right)}{64} = \frac{\pi \times (280^4 - 125^4)}{64} = 289{,}734{,}334 \text{ mm}^4$$

For cross-section C:

$$I = \frac{b_o {d_o}^3 - b_i {d_i}^3}{12} = \frac{150 \times 300^3 - 50 \times 200^3}{12} = \mathbf{304,166,666.7\ mm^4}$$

For cross-section D:

$$I = \frac{\pi D^4}{64} = \frac{\pi \times 275^4}{64} = 280{,}737{,}658.9\ \mathrm{mm}^4$$

For cross-section E:

$$I = \frac{bd^3}{12} = \frac{300 \times 225^3}{12} = 284{,}765{,}625\ \mathrm{mm}^4$$

Cross-section C has the largest 2nd moment of area.

11.

E. $245.8\ \mathrm{mm}$

[2 marks]

SOLUTION 11

2nd moment of area of a solid rectangular cross-section:

$$I = \frac{bd^3}{12}$$

Where, for a square, $b = d$,

$$\therefore I = \frac{b^4}{12}$$

$$\therefore b = \sqrt[4]{12I}$$

Substituting in the largest 2nd moment of area from Q7 ($337{,}500{,}000\ \mathrm{mm}^4$):

$$D = \sqrt[4]{12 \times 337{,}500{,}000} = \mathbf{245.8\ mm}$$

12.

E. No

[2 marks]

SOLUTION 12

Behaviour is assumed to be all elastic and therefore:

$$\frac{M_y}{I} = \frac{\sigma_y}{y}$$

where M_y is the moment required to cause yielding.

First yield will occur at $y = \pm\frac{d}{2}$, i.e. at the top and bottom edges:

$$\therefore M_y = \frac{\sigma_y \times I}{y} = \frac{\sigma_y \times \left(\frac{bd^3}{12}\right)}{\frac{d}{2}} = \frac{213 \times \left(\frac{245.8^4}{12}\right)}{\frac{245.8}{2}} = 115{,}312{,}500\ Nmm = 527.2\ kNm$$

Since $M < M_y$, yielding does not occur.

13.

D. $T = 185$ Nm, $P = 180$ N, $M = 210$ Nm

[2 marks]

SOLUTION 13

$$\tau = \frac{Tr}{J}$$

$$\sigma_{BM} = \frac{Mr}{I}$$

$$\sigma_A = \frac{P}{A}$$

$$\sigma_Z = \sigma_{BM} + \sigma_A$$

$$C = \frac{\sigma_Z + \sigma_\theta}{2} = \frac{\sigma_Z}{2}$$

$$R = \sqrt{\left(\frac{\sigma_Z - \sigma_\theta}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{\sigma_Z}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = C + R$$

	A	B	C	**D**	E
d	5.40E-02	5.40E-02	5.40E-02	**5.40E-02**	5.40E-02
T	175	175	195	**185**	285
P	175	185	160	**180**	170
M	185	195	175	**210**	125
sigma_1	1.80E+07	1.87E+07	1.79E+07	**2.00E+07**	1.78E+07

14.

A. 147.1 MPa

[2 marks]

SOLUTION 14

Given $\sigma_x = 35$ MPa, $\sigma_y = 115$ MPa and $\tau_{xy} = 60$ MPa:

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{35 + 115}{2} = 75.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{z\theta}^2} = \sqrt{\left(\frac{35 - 115}{2}\right)^2 + 60^2} = 72.1 \text{ MPa}$$

$$\sigma_1 = C + R = 75 + 72.1 = \mathbf{147.1\ MPa}$$

15.

A. 35.9 mm

[2 marks]

SOLUTION 15

$$K_I = 2.47\sigma\sqrt{\pi a}$$

$$\therefore K_{Ic} = 2.47 \times \frac{3}{5}\sigma_y\sqrt{\pi a_{cr}}$$

where

$$K_{Ic} = 106 \text{ MPa}\sqrt{\text{m}}$$

and

$$\sigma_y = 213 \text{ MPa}$$

Therefore,

$$106 = 2.47 \times \frac{3}{5} \times 213 \times \sqrt{\pi a_{cr}}$$

$$\therefore a_{cr} = \left(\frac{106}{2.47 \times \frac{3}{5} \times 213}\right)^2 \times \frac{1}{\pi} = 0.03589 \text{ m} = \mathbf{35.9\ mm}$$

16.

D. – 66.5 MPa

[2 marks]

SOLUTION 16

$$\frac{F}{A} = \frac{F_{init}}{A} - E\alpha\Delta T = \frac{2570}{100 \times 10^{-6}} - 72 \times 10^9 \times 20 \times 10^{-6} \times 64 = -6.65 \times 10^7\text{Pa} = \mathbf{-66.5\ MPa}$$

17.

C. 45.1 mm

[2 marks]

SOLUTION 17

For a shaft under pure torque, the Mohr's circle is centred on the origin and σ_1 and σ_3 will be the same magnitude and will also be the maximum allowable shear stress, therefore according to the Tresca yield criterion:

$$\tau_{max} = (\sigma_1 - \sigma_3)/2 = \sigma_y/2 = R$$

or

$$\frac{283}{2} = \tau_{max} = 141.5 \text{ MPa}$$

$$141.5 \times 10^6 = \frac{Tr}{J} = \frac{2T}{\pi r^3}$$

$$r = \sqrt[3]{\frac{2 \times 20500}{141.5 \times 10^6 \times \pi}} = \mathbf{0.0451 = 45.1\ mm}$$

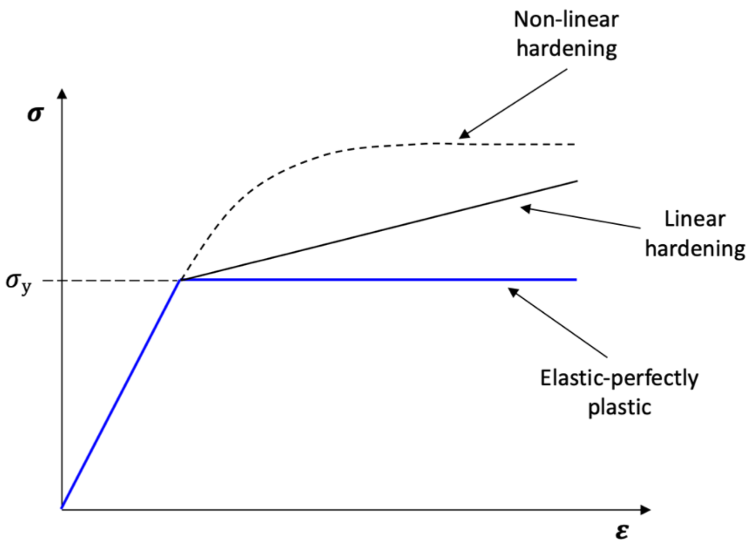
18.

B. Always less conservative than elastic-perfectly-plastic material behaviour

[2 marks]

SOLUTION 18

As shown in the figure below, once past the yield point, the curve of linear hardening material behaviour is consistently above that of the elastic-perfectly-plastic equivalent and is therefore less conservative.



19.

E. Linear hardening

[2 marks]

20.

C. Deviatoric stress

[2 marks]